Harnett County Schools
NC Math 4
Expanded Pacing Guide 2021-2022

Unit 1: Building Mathematical Community with Parent Functions & Key Features

5 Days

Standards: **Red = key vocabulary** **Green = verb**

Learning Intention A:

Develop positive attitudes about their ability to do mathematics**.**

- ❑ Students should **grow** to develop a growth mindset for problem-solving.

Learning Intention B:

Interpret functions that arise in applications in terms of the context.

- ❑ **NC.M3.F-IF.4: Interpret** key features of graphs, tables, and verbal descriptions in context to **describe** functions that arise in applications relating two quantities to include **periodicity** and **discontinuities**.

Learning Intention C:

Analyze functions using different representations.

- ❑ **NC.M3.F-IF.7: Analyze piecewise**, **absolute value**, **polynomials**, **exponential**, **rational**, and **trigonometric functions** (**sine** and **cosine**) using different representations to show key features of the graph, by hand in simple cases and using technology for more complicated cases, including: **domain** and **range**; **intercepts**; intervals where the function is **increasing**, **decreasing**, **positive**, or **negative**; **rate of change**; **relative maximums** and **minimums**; **symmetries**; **end behavior**; **period**; and **discontinuities**.
- ❑ **NC.M3.F-IF.9: Compare** key features of two functions using different representations by comparing properties of two different functions, each with a different representation (symbolically, graphically, numerically in tables, or by verbal descriptions).

EQ and Unpacking	Vocabulary Black = New Gray = Review	Resources
EQ A: • What is a mathematical growth mindset? • How can I use my mistakes as opportunities to learn? • What does it mean to be great at math? • What methods can I implement to start taking and maintaining responsibility for my own learning?	Piecewise Function Absolute Value Function Polynomials Exponential Function	• Mathematical norms poster from Jo Boaler's YouCubed website • Growth mindset resources from Jo Boaler's YouCubed website • Jo Boaler's Week of Inspirational Math Tasks - a collection of collaborative activities to promote and build classroom community • Believe in Yourself - video for developing a mathematical growth mindset **(LAUNCH)**

EQ B:
 	
• What mathematical patterns in tables, graphs, and symbolic rules are typical of the following functions: constant, linear, exponential, quadratic, square root, cubic, and absolute value?
 	
EQ C:
 	
• How can data tables, graphs, and rules relating variables be used to answer questions about relationships between variables?
• How can functions be used to model real-world situations?
• How do patterns and functions help us describe data and real-world physical phenomena?
• What are some properties and patterns of functions and their related parent functions?
 	
Unpacking C:
 	
• Students have identified the characteristics of graphs of other functions in previous math courses, including linear, quadratic, exponential, radical, and inverse variation functions.
  ○ Be familiar with the concept of intercepts, domain, range, intervals increasing/decreasing, relative maximum/minimum, and end behavior.
• Do not have to be able to algebraically manipulate a function in order to identify the key features found in graphs, tables, and verbal descriptions.
• Expected to use and interpret compound inequalities using inequality and interval notation to describe key features when appropriate.
• The representations of the functions should vary: table, graph, algebraical, or verbal description.
  ○ Comparing the key features should be the focus of the teaching for this standard, so the actual functions involved are not as important.
	Rational Functions
	
	Trigonometric Functions
	
	Relative Maximum
	
	Relative Minimum
	
	Increasing
	
	Decreasing
	
	End Behavior
	
	Periodicity
	
	Discontinuities
	
	Positive
	
	Negative
	
	Rate of Change
		• What does it mean to be great at maths? - activity for building classroom community **(LAUNCH)**
		• Skip Pattern - Week of Inspirational Math task to build classroom community through collaboration
		
		• **NC.M3.F-IF.4, NC.M3.F-IF.9 -** Common Parent Functions Reference Sheet
		• **NC.M3.F-IF.4, NC.M3.F-IF.9 -** Function Families
		• **NC.M3.F-IF.7-**Classroom Task: 4.1, 4.2, 4.6, 4.7 (Mathematics Visions Project-Secondary Mathematics III Module 4: Rational Functions)
		• **NC.M3.F-IF.7-**Classroom Task: 3.3 (Mathematics Visions Project-Secondary Mathematics III Module 3: Polynomial Functions)
		
		Formative Assessment Lessons/Task
		
		• **NC.M3.F-IF.4 -** Linear Marble Slides (Desmos)
		• **NC.M3.F-IF.4 -** Quadratic Marble Slides (Desmos)
		• **NC.M3.F-IF.4-** Exponential Marble Slides (Desmos)
		• **NC.M3.F-IF.4, NC.M3.F-IF.9 -** Which One Doesn't Belong (Graphs 16, 22, 27)
		• **NC.M3.F-IF.4, NC.M3.F-IF.9 -** Parent Functions Card Sort
		• **NC.M3.F-IF.4, NC.M3.F-IF.9 -**Parent Function Polygraph (Desmos)
		• **NC.M3.F-IF.4, NC.M3.F-IF.9 -** Inside Mathematics - Sorting Functions Activity
		• **NC.M3.F-IF.7-**Running Time (Illustrative Mathematics)

Unit 2: Functions

12 - 15 Days

Standards: **Red = key vocabulary Green = verb**

Learning Intention A:

Understand the properties and key features of **piecewise functions**.

- ❑ **NC.M4.AF.4.1**: **Translate** between algebraic and graphical representations of **piecewise functions** (**linear**, **exponential**, **quadratic**, **polynomial**, **square root**, **rational**, **radical**, **logarithmic**).
- ❑ **NC.M4.AF.4.2**: **Construct** a piecewise function to model a contextual situation.

Learning Intention B:

Apply properties of function composition to build new functions from existing functions.

- ❑ **NC.M4.AF.1.1**: Execute algebraic procedures to compose two functions.
- ❑ **NC.M4.AF.1.2**: Execute a procedure to determine the value of a composite function at a given value when the functions are in algebraic, graphical, or tabular representations.

Learning Intention C:

Understand how to model functions with **regression**.

- ❑ **NC.M4.AF.5.1**: Construct regression models of linear, quadratic, exponential, logarithmic, & sinusoidal functions of bivariate data using technology to model data and solve problems.
- ❑ **NC.M4.AF.5.2:** Compare residuals and residual plots of non-linear models to assess the goodness-of-fit of the model.

For Honors (From Pre-Calculus Unit 4)

Learning Intention D:

- ❑ **NC.PC.F.5.6: Implement** algebraic and graphical methods to find an **inverse function** of an existing function.
- ❑ **NC.PC.F.5.7: Conclude** by using **composition** to verify that one function is the inverse of another.

EQ and Unpacking	Vocabulary Black = New Gray = Review	Resources
EQ A: • What is a piecewise function? • In what ways can functions be built?	Function Piecewise	• **AF.4** Lumen Learning: Define and write Piecewise Functions **(Notes and Videos)** • **AF.4:** Math is Fun: Piecewise Functions **(Introduction)** • **AF.4:** Khan Academy: Piecewise Functions **(Introduction)**

Unpacking A: • Write piecewise functions based on real-world context. • Understand that each part of the given context may represent a different function type and understand that the domain for each piece is restricted by the context of the problem. • Use piecewise functions to model real-world situations (budgets, tax brackets, pricing, salaries, commission, parking rates, temperatures, velocities, growth charts, roller coasters, etc...). • The use of step functions in piecewise functions could also be an honors extension if desired. **EQ B:** • What is a composite function? **Unpacking B:** • Extend their understanding of function notation and evaluating functions to working with compositions of two functions. • Evaluate a composition of functions for specific values in the domain. • Understand the net effect of $f \circ g$ is $a \to g(a) \to f(g(a))$ for any real value, a. Understand that a is in the domain of g and that $g(a)$ is in the domain of f. • No limit as to the type of functions to be used when composing functions. • In problems involving roots in the denominator, it is not necessary to rationalize the denominator. • NOT intended for students to use compositions to assess inverse functions; however, this could become an extension for honors students if there is time available since they have discussed inverse functions in Math 3. • Apply evaluated functions for a specific value in the domain, given a function in algebraic, graphical, or tabular form, to compositions of functions in a variety of representations. This includes functions presented in two different representations.	Linear Exponential Quadratic Polynomial Square Root Rational Radical Logarithmic Domain Range Input Output Independent Dependent **Composite Function** **Interpolation** **Extrapolation**	• **AF.1.1** Math is Fun: Composite Functions **(Notes)** • **AF 5.1, AF. 5.2** Khan Academy: Scatterplots and Correlation Review • **AF 5.1, AF. 5.2** Creating Regressions using Desmos calculator • **AF 5.1, AF. 5.2** Creating Regressions using TI 84 • **AF 5.1** Alligator Investigation (Desmos) • **AF 5.2** Line of Best Fit (Desmos) **(Launch Residuals)** • Honors: **PC.F.5.6, PC. F.5.7 -** Inverse Functions and Composite Connections **- Textbook containing examples of inverse and composite inverses.** Formative Assessment Lessons/Task • **A.F.4** Piecewise Functions Exploration **(Desmos)** • **AF 1.1 and AF.1.2** Practice with Composition of Functions • **AF 1.1, AF1.2 Quizizz-**Quiz:Alg2 Composite Functions • **A.F.5** Choosing a Regression Model • **AF. 5.1, AF5.2 Quizizz-**Scatterplots and Correlation Coefficient • Honors: **NC.PC.F.5.6 -** Functions and their Inverses **- Mathematics Vision Project (Uses function machines to model functions and their inverses. Focuses on finding inverse functions and verifying that two functions are inverses.)** • Honors: **NC.PC.F.5.7-** Composition and Inverse (Section 2)

EQ C: ● When does a function best model a situation? ● How can recognizing key features of graphs help solve problems more efficiently? ● How can identifying a pattern or structure help the solution process? ● How can I communicate the properties of a data set to illuminate its important features? **Unpacking C:** ● Use regression to model bivariate data. ○ Includes quadratic, logarithmic, and sinusoidal functions (more focus on logarithmic and sinusoidal functions in their particular unit). ○ Use regression models to solve real-world and mathematical problems. ● As statistical thinkers, students need to understand part of the process when exploring data by first trying to get a look at what the data is doing. ○ Create a scatterplot of the data and try to describe the overall trend they are seeing. ○ Decide what types of functions might be appropriate to model the data. ○ Try to fit a model. Can be done on a graphing calculator or through other statistical technology. ○ Must then decide how appropriate that model is for the data. ○ It is extremely important for students to realize, in the statistical process, that this is part of explaining why and convincing others that their model is a good fit. ● Residuals and residual plots are further evidence students will be using to justify that they have found the best fit model because r and r^2 do not apply as measures of goodness-of-fit to all functions. ○ Video - r^2 explained: https://www.youtube.com/watch?v=IMjrEeeDB-Y	**Scatter plot** **Independent variable** **Dependent variable** **Linear** **Quadratic** **Exponential** **Data set** **Outliers** **Bivariate data** **Direction** **Form** **Models** **Correlation coefficient** **Linear model of best fit** **Regression analysis (r and r^2)** **Scale**	

○ Apply knowledge from Math 1 about interpolating and extrapolating and validity of predictions. ○ Closely analyze the residual plots of non-linear models to determine the goodness-of-fit of a selected non-linear model. ○ Understand that a residual plot with no discernable pattern indicates the selected model is a good fit for the data set. **Honors EQ D:** ● Why is it important to remember to restrict the domain of a function at times when determining a one-to-one relationship? ● How do you determine if two functions are inverses of each other algebraically? **Unpacking D:** ● Students will extend their understanding of the inverse relationship to polynomial and rational functions, as well as the six trigonometric functions. ● Build an understanding of a function being one-to-one using the horizontal line test. ● Build on their ability to restrict the domain of the original function in order for this function to be one-to-one. ● Find an inverse graphically and algebraically. ● Use composition to verify that two functions are inverses of each other.	**Quantitative variable** **Significance**	

Unit 3: Logarithmic Functions

8 Days

Standards: **Red = key vocabulary Green = verb**

Learning Intention A:

Apply the properties and key features of **logarithmic functions**.

- ❑ **NC.M4.AF.3.1**: **Execute** properties of logarithms to simplify and solve equations algebraically.
- ❑ **NC.M4.AF.3.2**: **Implement** properties of logarithms to solve equations in contextual situations.
- ❑ **NC.M4.AF.3.3**: **Interpret** key features of a logarithmic function using multiple representations.

Learning Intention B:

Understand how to model functions with **regression**.

- ❑ **NC.M4.AF.5.1**: **Construct** regression models of linear, quadratic, exponential, logarithmic, and sinusoidal functions of bivariate data using technology to model data and solve problems.

EQ and Unpacking	Vocabulary Black = New Gray = Review	Resources
EQ A: • How can we apply properties of logarithms to solve equations? Explain. • How do we use the properties of logarithms to expand or condense expressions? • How do we utilize logarithms to solve real-world scenarios? **Unpacking A:** • Build upon students' understanding of the properties of exponents when applying the properties of logarithms. • Require students to procedurally rewrite logarithmic expressions to solve logarithmic equations; including natural log and *e*. • Solve logarithmic equations based on contextual situations. • Determine when it is appropriate to apply logarithmic properties when presented with a problem based on a contextual situation.	Expand Condense Product property Quotient Property Power Property Parts of log and exponential Equation Simplify Domain	• **AF.3.1, AF.3.2, AF.3.3** Logs Starter **(Launch)** • **AF.3.1, AF.3.2, AF.3.3** Logarithms **(Launch & Lesson)** • **AF.3.1** Simplify Logarithms Video (YouTube) • **AF.3.1** Properties of Logarithms Notes (Montgomery Institute) • **AF.3.2** Applications of Logarithms Video (YouTube) • **AF.3.2** Applications of Logarithms Video (YouTube) • **AF.3.3** Logarithmic Functions Notes (MathBits) • **AF.3.3** Logarithmic Functions Notes, Videos, Practice (CK-12) • **AF.5.1** Regression Examples Video (YouTube) **(Launch for Regression)** ○ **Can use one of these examples as a Launch Activity. Open up the lesson by recalling regression and determining which would be the "best fit."** • **AF.5.1** Regression Notes (CK-12)

○ May include: creating an exponential equation and then solving the equation using properties of logarithms. ● Understand and interpret key features (intercepts, end behavior, domain, range, and intervals where the function is increasing and decreasing) of logarithmic functions. **EQ B:** ● How do we determine which regression model is best for our data? ● What are the limitations of a particular function when modeling a real-world scenario? ● How do you model a quantity that changes regularly over time by the same percentage? **Unpacking B:** ● Apply knowledge from Math 1 about interpolating and extrapolating and validity of predictions. ● Closely analyze the residual plots of non-linear models to determine the goodness-of-fit of a selected non-linear model. ● Understand that a residual plot with no discernable pattern indicates the selected model is a good fit for the data set.	**Range** **Increasing** **Decreasing** **X-intercept** **Y-intercept** **End behavior** extrapolation Interpolation Regression Analysis Residual Good fit **Bivariate data**	● **AF. 5.1** https://mathbits.com/MathBits/TISection/Statistics2/logarithmic.htm Formative Assessment Lessons/Task ● **AF.3.1** CK-12 Lesson & Practice ● **AF.3.1** Interactive Solving Log Equations: Open Middle **(Could be Honors level thinking questions)** ● **AF.3.1** Quizlet Solving Logarithmic Equations ● **AF.3.1** Solving Logarithmic Equations Desmos Activity (Teacher Guide) ● **AF.3.2, AF3.3** Exploration of Logs (ProjectMath) ● **AF.3.3** Comparing Exponential & Logs Graphs (Desmos) ● **AF.3.3** Graphing Logarithmic Functions (CK-12) ● **AF.5.1** Line of Best Fit Worksheet With Answers

Unit 4: Trigonometry

15 - 20 Days

Standards: **Red = key vocabulary** **Green = verb**

Key Features:

Learning Intention A:

Apply properties of **trigonometry** to solve problems.

- ❑ **NC.M4.AF.2.1: Translate** trigonometric expressions using the **reciprocal** and **Pythagorean identities**.
- ❑ **NC.M4.AF.2.2: Implement** the **Law of Sines** and the **Law of Cosines** to solve problems.
- ❑ **NC.M4.AF.2.3: Interpret** key features (**amplitude**, **period**, **phase shift**, **vertical shifts**, **midline**, **domain**, **range**) of models using sine and cosine functions in terms of context.

For Honors:

- ❑ **NC.M4.AF.2.1 and 2.2 Extension:**
  - ❑ **NC.PC.F.2.2(HONORS): Explain** the relationship between the **symmetry** of a **unit circle** and the **periodicity** of **trigonometric functions**.
- ❑ **NC.M4.AF.2.3 Extension:**
  - ❑ **NC.PC.1.2(HONORS): Interpret** algebraic and graphical representations to determine key features of **tangent**, **cotangent**, **secant**, and **cosecant**.

Learning Intention B:

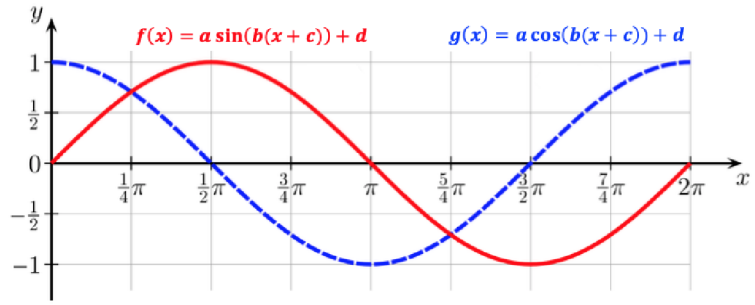
Understand how to model functions with **regression**.

- ❑ **NC.M4.AF.5.1:** Construct regression models of linear, quadratic, exponential, logarithmic, & **sinusoidal functions** of bivariate data using technology to model data and solve problems.

EQ and Unpacking	Vocabulary Black = New Gray = Review	Resources
EQ A: • How can the Reciprocal and Pythagorean Identities be used to simplify other trigonometric expressions? • What methods can be used to write trigonometric equations?	Tangent X coordinate Y coordinate Opposite Side Adjacent Side	• **LAUNCH: Why is Trigonometry Important - Unit Launch Video** • **AF.2.1: The Pythagorean Identities- Notes** • **AF.2.1 AND 2.3: Trigonometric Functions and Identities- Multiple Lessons** • **AF.2.1 AND 2.3: Transformation Quiz-Quiz**

• How does the given information and/or structure of the equation influence the selection of an efficient method for writing trigonometric equations?
• How are the properties and understandings used to transform trigonometric equations?
• What connections exist between corresponding trigonometric equations?
• How can the Law of Sines and Cosines formulas be used to model and solve real-world situations?
• How do you construct and apply transformations to the Sine and Cosine Functions? (Honors: Extend to Tangent, Cotangent, Cosecant, and Secant functions)
• How can Sine and Cosine Functions be used to model real-world situations?
 	
Unpacking A:
 	
• First time students will encounter trigonometric identities.
  ○ Use algebraic reasoning to rewrite trigonometric expressions in simplified equivalent forms using the reciprocal and Pythagorean identities.
• Use the Law of Sines and the Law of Cosines to find the unknown measurements in right and non-right triangles. They should be able to:
  ○ Distinguish between situations that require the Law of Sines (ASA, AAS, SSA) and situations that require the Law of Cosines (SAS, SSS);
  ○ Represent real-world problems with diagrams of non-right triangles and use them to solve for unknown side lengths and angle measures;
  ○ Solve for missing side lengths and angles using Law of Sines and Law of Cosines.
  ○ NOTE: *The ambiguous case for oblique triangles is NOT an expectation in NC Math 4.*
• Students should know how to transform other functions so they can apply the "same rules" for shifts, stretches, and compressions to the sine and cosine graphs.
	Included Angle
	Bearing
	Angle of elevation
	Angle of depression
	SSS
	AAS
	SAS
	Graph
	Domain/Range
	Period
	Amplitude
	Phase Shift
	Vertical Shift
	Midline
	Coefficients
	Intercepts
	X Coordinate
	Y Coordinate
	Regression Model
	Diagnostics on/off
	Best fit model
	Residuals
		• **AF.2.1: Proving Trig Identities Cut and Paste- Pairing or Bell Ringer Activity**
		• **AF.2.1: Trig. Identity Proofs- TI 84 Activity**
		• **AF.2.1: Trig. Identities - Desmos Activity**
		• **AF.2.2(Honors): Deriving the Law of Cosines- Exploration Activity(pg. 1)**
		• **AF.2.2: Which one does not belong? - Task/Launch/Bell Ringer(shape 23)**
		• **AF.2.2: When to uses Law of Sines and Cosines- Lesson**
		• **PC.F.2.2(Honors): Unit Circle Game-Activity**
		• **AF.2.3: Write the equation of Sine and Cosine Graphs-Quiz/Worksheet**
		• **AF.2.3: Marbleslides:Periodics -Desmos Activity**
		• **AF.2.3: Polygraph: Periodic Functions - Desmos Activity**
		• **AF.2.3: Graphing Trigonometric Functions Activity-Activity/Formative Assessment Task**
		• **AF.2.1,2.2,2.3: Worksheets- Worksheet/HW/Classwork**
		• **AF.5: Sine Regression Worksheet - Worksheet**
		• **AF.5: Sinusoidal Functions Activity- Desmos Activity**
		
		Formative Assessment Lessons/Task
		
		• **AF.2.1: Trigonometric Functions 1- Short Task**
		• **AF.2.1 AND 2.3: Which one does not belong? - Task/Launch/Bell Ringer(Graph 21 and 23)**
		• **AF.2.2: Law of Cosine-Performance Task(last pg.)**
		• **AF.2.2: Which one does not belong? - Task/Launch/Bell Ringer(shape 23)**
		• **AF.2.3(Honors): Representing Trigonometric Functions- Formative Assessment Lesson**
		• **AF.2.3: Graphing Trigonometric Functions Activity -Activity/Formative Assessment Task**
		• **AF.2.3: Which one does not belong? - Task/Launch/Bell Ringer(Graph 21 and 23)**
		• **AF.2.3 AND AF.5: Apply Trig to new situations(Sine Curve)-Performance Task(pg.6-9)**

- Translate between different representational forms of trigonometric models (i.e. graphs, tables, and algebraic formats).
- Phase shift for trigonometric functions is a new concept.
- Understand the relationship between the sine and cosine functions in relation to their key features.
- Determine the meaning of variables and coefficients of trigonometric models (i.e. graphs, tables, and algebraic formats).
- The standard for sine and cosine functions:
  $f(x) = a\ sin\ (b(x + c)) + d$
  $g(x) = a\ cos\ (b(x + c)) + d$



where *a, b, c,* and *d* are constants.

Honors - Unpacking A:

- Develop an understanding of using the trigonometric ratios and the coordinates of the unit circle to determine the values of sine, cosine, tangent, and their reciprocal functions.
- Use their understanding of a period of a sine function to extend to periods of cosine, tangent, cotangent, secant, and cosecant.
  - Make the connection of lines of symmetry, repeating coordinates on the unit circle, and periods of the functions.
- The periodicity of trigonometric functions shows the identities:
  - $sin(\alpha + k \cdot 2\pi) = sin(\alpha)$ and $cos(\alpha + k \cdot 2\pi) = cos(\alpha), k \in Z$
  - $tan(\alpha + k \cdot \pi) = tan(\alpha)$ and $cot(\alpha + k \cdot \pi) = cot(\alpha), k \in Z$

Line of best fit

Unit Circle

Terminal Ray

Reference Angle

Symmetry

Radian

Degrees

Asymptote

Vertical Asymptote

Holes

Discontinuity

Pythagorean Identities

Reciprocal Identities

Secant

Cosecant

Cotangent

6 Trig Ratios

Law of Sines and Cosines

Sinusoidal

• Determine and interpret key features previously studied in Math 3 to include the vertical asymptotes for tangent, cotangent, secant, and cosecant functions.
  ○ Not expected to find asymptotes from the algebraic expression of trigonometric functions.
  ○ Only expected to describe phase shift and change of period for sine and cosine.
 	
EQ B:
 	
• How can technology be used to create a regression model of a Sinusoidal Function that models a set of data and solves real-world application problems?
 	
Unpacking B:
 	
• Construct regression models for quadratic, logarithmic, and **sinusoidal** functions of bivariate data.
• Use regression models to solve real-world and mathematical problems.		

Unit 5: Exploratory Data Analysis

10 - 15 Days

Standards: **Red = key vocabulary** **Green = verb**

Key Features:

Learning Intention A:

Create statistical investigations to make sense of real-world phenomena.

- ❑ **NC.M4.SP.1.1: Construct** statistical questions to guide explorations of data in context. *[For example, a question that: anticipates **variability**, is answerable with data, states the **population** under consideration, states the attribute under investigation, and is clear enough to guide the analysis of the data]*
- ❑ **NC.M4.SP.1.2: Design sample surveys** and **comparative experiments** using **sampling methods** to **collect** and **analyze** data to answer a statistical question.
- ❑ **NC.M4.SP.1.3: Organize** large datasets of real-world contexts (i.e. datasets that include 3 or more measures and have sample sizes >200) using technology (i.e. spreadsheets, dynamic data analysis tools) to determine: types of variables that are in the data set, possible outcomes for each variable, statistical questions that could be asked of the data, and types of numerical and graphical summaries could be used to make sense of the data.
- ❑ **NC.M4.SP.1.4: Interpret** non-standard data visualizations from the media or scientific papers to make sense of real-world phenomenon. *[This standard is not referring to traditional graphs such as a histogram, dot plot, boxplot, bar graph, pie chart, or stem and leaf plot]*

EQ and Unpacking	Vocabulary Black = New Gray = Review	Resources
EQ A: • What are the key properties of a dataset? • Why is exploratory data analysis important? • How can predictions be made based on data? • What evidence do we need to know to make sense of the data and to analyze that accurately? • How can I construct unbiased questions and sampling methods to accurately analyze data? • How can technology be used to analyze data? • How can data be analyzed and compared? • How do people use data to influence others? • How can I interpret visualizations of data from a variety of sources and formats?	Sample Population, experiment, parameter, random Sample Observations Surveys Spreadsheet **Individuals,**	• **SP.1** PBS Categorical & Quantitative Data (Launch Videos) • **SP.1** CK12 Flexbook Chapters One, Two, & Six • **SP.1.1** How to Formulate Questions (Launch) • **SP.1.1** Corona Virus Article (Launch) • **SP.1.1** Supply Chain • **SP.1.1** Checklist for Exploratory Data Analysis • **SP.1.1** CPALMS Lesson on Variability • **SP.1.2** CPALMS Population Sampling & Beekeeping, Legal Catch Rates for Fish, Choosing a New School Advisory Panel (many more) • **SP.1.2** Illustrative Math Identifying Statistical Questions • **SP.1.2** CK12 Sampling • **SP.1.2** Texas Instruments Design Activities • **SP.1.3** Illustrative Math How Many Leaves on a Tree? • **SP.1.3** Article about Challenges

• How do you communicate your results so there is common understanding? • How does the way I collected data affect the conclusions that can be drawn? • How do I use statistics to explain the variability and randomness in a set of data? **Unpacking A:** • The GAISE Report for Statistics outlines a four-step process for problem-solving as students engage in statistical inquiry: 1. Formulating a statistical question that anticipates variability and can be answered by data. 2. Designing and implementing a plan that collects appropriate data. 3. Analyzing the data by graphical and/or numerical methods. 4. Interpreting the analysis in the context of the original question. • The first step of the statistical problem-solving process - students construct statistical questions that "anticipate an answer based on data that vary". • The second and third steps of the problem-solving process - design surveys and comparative experiments to plan the collection and analysis of the data stemming from the original statistical question. • Develop their understanding of sampling methods that are both biased and unbiased in relation to surveys and comparative experiments. • Combine their understanding of how to construct statistical questions and different sampling methods to create surveys and design experiments to answer statistical questions. • Given real-world context - design an experiment based on random sampling to investigate the phenomenon. ○ Understand unbiased probability sampling methods (e.g. SRS, stratified, cluster, systematic, multistage) ○ Use appropriate methods when designing experiments or conducting surveys. ○ Aware of biased sampling methods and avoid them when collecting data.	**variables,** **population** **Variability** **dynamic data analysis tools** **Non-standard data visualizations**	• **SP.1.3** Various Tools • **SP.1.3** Desmos Activity • **SP.1.4** Blog with Examples of Data Visuals (there is a tool to create your own as well, but you do have to sign up) • **SP.1.4** Historical Examples • **SP.1.4** Examples (Data Visualization) Formative Assessment Lessons/Task • **SP.1.1** Mathshell Lesson • **SP.1** Khan Academy Data Analysis Unit • **SP.1** Choosing a Study Design for the Polio Vaccine • **SP.1** How Often Do I Say the Word "um"?

• Build on the understanding that more sampling may yield more accurate results, students will now work with larger data sets and multiple measures.
  ○ Require the use of technology that can efficiently output summary measurements and create graphical displays of data for very large data sets. This may include, but is not limited to, statistical software, spreadsheets, and online analytical tools:
  ■ Spreadsheet software, such as Google Sheets and Microsoft Excel can be used at the very minimum.
  ■ CODAP is a free open-source software for data analysis.
  ■ Cause Web also has a variety of data collection and analysis tools listed on its site.
• Students understand the display, what information it conveys, and how to translate the information into another form (e.g. a table, another graph, etc.).		

Unit 6: Probability Distributions

10 Days

Standards: **Red = key vocabulary** **Green = verb**

Learning Intention A:

Apply probability distributions in making decisions in uncertainty.

- ❑ **NC.M4.SP.3.1: Implement discrete probability distributions** to model a random phenomenon and **make** decisions (for example, **expected value** of playing a game, etc)
- ❑ **NC.M4.SP.3.2: Implement** the **binomial distribution** to model situations and **make** decisions.
- ❑ **NC.M4.SP.3.4: Implement** the **normal distribution** as a **probability distribution** to determine the likelihood of events occurring.

EQ and Unpacking

EQ A:

- What is an expected value and how do I use this knowledge to make informed decisions?
- Why is it important and intriguing to know the probability of an occurrence?
- In what ways is a binomial distribution different from a probability distribution and how can knowing how to find the probability of these help in predicting future events?
- How do you represent, interpret and apply properties of normal distributions?
- How is the normal distribution curve used in real life?

Unpacking A:

- Discrete probability distributions are based on **discrete random variables**, where there is a finite number of possible outcomes.
- A **discrete probability distribution** lists each outcome and its associated probability.

Vocabulary

Black = New
Gray = Review

Variable

Probability

Histogram

Discrete Probability

Distribution

Continuous

Probability Distribution

Theoretical Probability

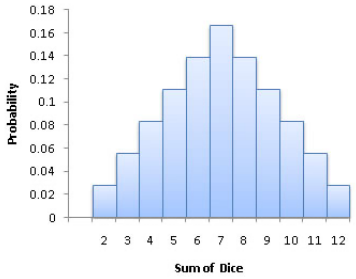
Experimental Probability

Simulation

Infinite

Resources

- **SP.3** PBS Making Decisions With Probability (Launch Videos)
- **SP.3.1** Would You Rather Math Dice, Coins (Launch)
- **SP.3.1** A Sweet Task (Launch)
- **SP.3.1** Illustrative Math Tasks (Bob's Bagel Shop, Fred's Fun Factory, Sounds Really Good, Sort of…)
- **SP.3.1** Robert Kaplinsky (Video Poker Lesson)
- **SP.3.1** CK12 Expected Value Play Learn Interact eXplore
- **SP.3.2** Math Is Fun Binomial Distributions
- **SP.3.2** CK-12 Lessons (multiple lessons)
- **SP.3.2** Binomial Probability Practice
- **SP.3.2** Binomial Distribution (Video that explains what a Binomial Distribution is and how the binomial formula is used (gives a pretty detailed explanation but is somewhat dry. I wouldn't necessarily show it to students, but it can teach the teacher.)
- **SP.3.2** How to do Binomial PDF and CDF on a TI-84 (Video, does not explain what to do when you need to find X>n)
- **SP.3.2** Mathematics Vision Project (Probability Unit)
- **SP.3.4** How to use the Empirical Rule (Video)

○ Create a probability distribution table listing each outcome and its probabilities. • Understand that a probability distribution can be used to find the expected value or average weighted outcome of discrete random variables. • Understand the expected value in determining the fairness of a game. • A **probability histogram** is a graphical display that graphs each outcome on the horizontal axis and its probability on the vertical axis. ○ For example, the probability histogram on the right represents the sum of the rolls of 2 six-sided dice and the probability of obtaining each sum. The probabilities in a discrete probability distribution should add to 1.  • Students understand the four main criteria for a binomial setting: 1. A fixed number of observations, n. 2. Each observation (n) is independent. 3. Each observation falls into one of two possible categories, known as success or failure. 4. The probability of success (p) is the same for each observation. • Understand that the distribution of the count of X successes in the binomial setting is called the binomial distribution with parameters n and p where ***n*** represents the number of observations and ***p*** represents the probability of success; abbreviated as X is $B(n, p)$. • Understand that when n is large, the distribution of X is normal, $N(np, \sqrt{np(1-p)})$ and the normal approximation can be used when n and p satisfy the following conditions: 1) $np \geq 10$ and $n(1-p) \geq 10$.	**Finite** **Outcomes** **Independent** **Dependent** **Trials** **Expected Value as Mean/Average/Weighted Average Uniform Random Variable** **Binomial Random** **Binomial Probability Distribution** **Binomial Random Variable** **Binomial Experiment Trial** **n (number of observations)** **p (probability of success)** **Standard Deviation** **Simulation** **Bell Curve**	• **SP.3.4** Z-scores and Normal Distributions (Video) • **SSP.3.4** Using TI-84 for NormalCDF and invNorm (Video, SUPER Important since we don't want the teachers to use the Z-table) • **SP.3.4** Math Is Fun False Positives & False Negatives (discussion & examples) • CK12 Normal Distribution Play Learn Interact eXplore • **SP.3** Geogebra Probability Distribution Activities • **SP.3** Quizizz Online Practice Sets • **SP.3** DeltaMath search "probability" • **SP.3** Better Lesson Using Probability to Make Decisions (multiple resources) • **SP.3** IXL Online Practice (Subscription for students required) • **SP.3** Illuminations (Membership required) • **SP.3** Desmos Probability Review Formative Assessment Lessons/Task • **SP.3.1** Bob's Bagel Shop • **SP.3.1** Would You Rather? • **SP.3.2** Khan Academy Random Variables • **SP.3.4** Khan Academy Normal Distribution

• Know when it is appropriate to use the normal distribution and how to determine probabilities using the normal distribution. • Know how to find an observation from a given probability based on an assumption of the normal distribution. • Understand that the normal curve is a density curve, where the area under the curve is exactly one and above the horizontal axis. • Understand the empirical (68-95-99.7) rule and how it applies to the normal curve. • Know that the standard normal distribution $N(\mu, \sigma)$ has mean, μ, and standard deviation, σ. • Understand z-scores and know how to find the standardized value of x using $z = \frac{x-\mu}{\sigma}$, where z represents the distance that a value is from the mean, μ, in standard deviation, σ, units. *Technology use is at the discretion of the teacher.	**Continuous** **Distribution** **Random Variables** **Normal Probability Distribution (Normal Curve) Continuous** **Random Variables Z-scores (standardized scores)** **Percentiles** **Empirical Rule** **Density Curve** **Mean** **Standard Deviation**	

Unit 7: Statistical Inference

15 Days

Standards: **Red = key vocabulary** **Green = verb**

Learning Intention A:

Apply informal and **formal statistical inference** to make sense of, and make decisions in, meaningful real-world contexts.

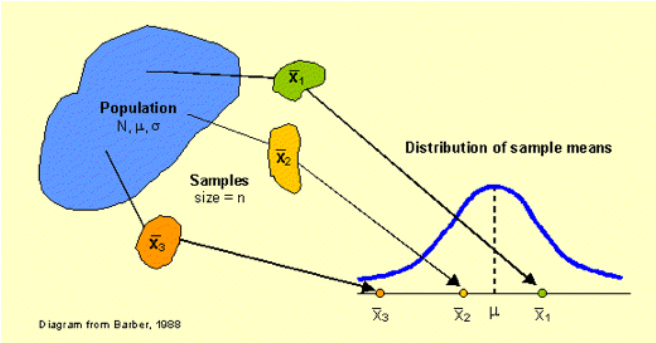
- ❑ **NC.M4.SP.2.1: Design** a **simulation** to create a **sampling distribution** that can be used in making **informal statistical inferences**.
- ❑ **NC.M4.SP.2.2: Construct confidence intervals** of **population proportions** in the context of the data.
- ❑ **NC.M4.SP.2.3: Implement** a **one-proportion z-test** to determine if an **observed proportion** is significantly different from a **hypothesized proportion**.

Learning Intention B:

Apply probability distributions in making decisions in uncertainty.

- ❑ **NC.M4.SP.3.3: Recognize** from **simulations** of **sampling distributions** of **sample proportions** that a **normal distribution** can be used as an approximate model in certain situations.

EQ and Unpacking	Vocabulary Black = New Gray = Review	Resources
EQ A: • How can technology be used to analyze data? • How can data be analyzed and compared? • How can predictions be made based on data? • How do people use data to influence others? • How can we base decisions on chance? • How can probability be used to simulate events and to predict future happenings? • What are the benefits of simulating events as opposed to gathering real data? • How can you determine whether a sample accurately represents the population? • How can I find the sampling distribution of a sample proportion? • How can I find the sampling distribution of a sample mean?	Sampling distribution Population Random Sample Inferential Statistics Sample Simulation normal distribution **Parameter** **Margin of Error** **Confidence interval**	• **SP.2** PBS Inferences & Conclusions (Launch) • CK12 Flexbook Chapters Four, Five, & Eight • Against All Odds (Launch Videos) • Data Tells Us About Ourselves (Launch) • The CDC is lumping positive COVID-19 viral and antibody tests together. Here's why that's bad. (Launch, article for discussion) • Introduction to Data Science Units (project for Honors needs an extension for analysis) • Using Data: Testing a New Product (Launch) • **SP.2** Geogebra Statistics Activities • **SP.2** Delta Math search "Statistics" (multiple activities for the entire unit - must create a teacher account) • **SP.2.1** TI Calculator Activity - Simulation • **SP.2.1** StatTrek Simulation Instructions • **SP.2.1** Illustrative Mathematics Tasks

• How do I use theoretical and empirical results to determine if a treatment was effective? • How does the way I collected data affect the conclusions that can be drawn? • How do I use statistics to explain the variability and randomness in a set of data? • How do I interpret the margin of error of a confidence interval? • How do I use a margin of error to find a confidence interval? **Unpacking A:** • Formal statistical inference is new. • Design a simulation and use the information to create a sampling distribution to make informal inferences. • Understand that a sampling distribution is a graph of a statistic generated from multiple samples of the same size from the population of interest (see figure below).  • Understand the connection between repeated sampling and the construction of the confidence interval. ○ Yates, Moore, and Starnes (2002) state that "**a confidence interval** uses sample data to estimate an unknown population parameter with an indication of how accurate the estimate is and of how confident we are that the result is correct." • Use $\hat{p} \pm z * \sqrt{\frac{p(1-p)}{n}}$ to construct the confidence interval. ONLY construct confidence intervals for population proportions only.	**Z-test** **observed proportion** **hypothesized proportion** **sample proportions**	• **SP.2.2** Math Is Fun Confidence Intervals (discussion & examples) • **SP.2.2** TI Calculator Activities - Confidence Intervals • **SP.2.2** Find confidence intervals for population proportions | IXL (online practice with Algebra 2 objectives strand EE) • **SP.2.2** Hypothesis Testing Worksheet • **SP.2.3** Hypothesis Test for Difference in Proportions Video (Khan Academy) • **SP.2.3** Constructing Hypotheses for Two Proportions Video (Khan Academy) • **SP.2.3** Writing Hypotheses for Testing the Difference of Proportions Practice (Khan Academy) • **SP.2.3** Hypothesis Test for Difference in Proportions Example Video (Khan Academy) • **SP.2.3** Test Statistic in aTwo-Sample z-test for the Difference of Proportions Practice (Khan Academy) • **SP.2.3** P-value in a Two-Sample z-test for the Difference of Proportions Practice (Khan Academy) • **SP.2.3, SP.3.3** ck12.org statistics (multiple lessons) • **SP.2.3** Normal Distribution Practice • **SP.3.3** Normal distributions | Khan Academy (review of Normal Distribution) • **SP.3.3** Covid Data USA • **SP.3.3** MathShell Statistics (multiple lessons) • **SP.2, SP.3** AP Central Resources as well as Sampling and Inference Activities • **SP.2 SP.3** Would You Rather? (SAT Score) • **SP.2** Quizizz Hypothesis Testing Worksheet, Confidence Intervals Practice, Simulations • **SP.2, SP.3** Better Lesson Making Inferences & Justifying Conclusions • NC2ML - Applet for Simulations • NC2ML - CODAP (works well for simulating repeated sampling for creating a sampling distribution) • Resources for Teaching K-12 Statistics (many good resources specific to simulation see pages 32 and 33)

• Understand all constants and coefficients of the formula and can interpret confidence intervals in the context of the problem. • Understand the conditions by which it is appropriate to use a sample to construct a confidence interval for a given sample. • Calculation of the margin of error becomes formalized; extending from informal understanding based on simulation in NC Math 3. • **Hypothesis testing** is the use of statistics to determine the probability that a given hypothesis is true. ○ Understand hypothesis testing as a formal statistical procedure for determining the true population parameter; in this case, students will be determining the probability that a sample proportion is the true population proportion based on a one-proportion z-test. In order to fully understand the process of hypothesis testing, students should engage in the entire process and relate each step to the context of the problem. A basic process includes the following steps: ***Step 1:*** *State the null and alternative hypotheses using symbols and in the context of the problem.* $H_0 : p = p_0$ $H_a : p \neq p_0$ **or** $H_a : p < p_0$ **or** $H_a : p > p_0$ ***Step 2:*** *Verify the conditions of the inference procedure (one-proportion z-test).* *If* $np_0 \geq 10$ *and* $n(1 - p_0) \geq 10$*, then we can use the normal distribution* ***Step 3:*** *Carry out the procedure by hand or using technology. Find the z-statistic and the corresponding p-value in the z-table* $z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$ ***Step 4:*** *Interpret the results in the context of the problem; stating whether to reject or to fail to reject the null hypothesis. Measure statistical significance against a predetermined significance level (0.05 unless otherwise stated).* • *If* $p - value \leq \alpha$*; reject* H_o*: the difference between the proportions is statistically significant.* • *If* $p - value > \alpha$ *;fail to reject* H_o*: the difference between the proportions is NOT statistically significant.* **EQ B:**		• NC2ML - PBS Crash Course to understanding Confidence Intervals • Explore Coverage on Confidence Intervals (interactive site to explore confidence intervals) • NC2ML - Friday Institute has 2 Massive Open Online Courses that can be taken for free online that help teach statistics conceptually Teaching Statistics through Inferential Reasoning and Teaching Statistics through Data Investigations Formative Assessment Lessons/Task • **SP.2** Study Design | Khan Academy (Study Design) • 1 in 6 Wins (activity is an introduction to the thinking behind a one proportion z test) • **SP.3** Over the Hill - Aging on a Normal Curve • **SP.3** What Does Normal Distribution Sound Like?

- How do I decide if the normal distribution describes a set of data?
- When do I use the normal distribution to estimate probabilities?

Unpacking B:

- Understand that a sampling distribution is a graphical display of a μ statistic for sample data.
- Understand that the sampling distribution is a probability distribution that is obtained from the collection of a large number of samples of the same size.
    - It represents a wide range of possible outcomes of a statistic for a given population.
- Understand that there is less variability in larger sample sizes than in smaller sample sizes (less than 30).
    - So, the sampling distribution becomes more *like* the normal distribution as the sample size increases and more samples are drawn.
- Understand that they can use the normal approximation $N(p, \sqrt{\frac{p(1-p)}{n}})$ for the probability distribution of **sample proportions** when:
    1) The population is at least 10 times as larger as the sample
    2) $np > 10$ and $n(1-p) > 10$
- Understand that they can use the normal approximation $N(\mu, \frac{\sigma}{\sqrt{n}})$ for the probability distribution of **sample means** when the population is at least 10 times as larger as the sample

Common Misconceptions

- The belief that hypothesis tests are deterministic or give absolute proof (cause & effect).
- Confusion about the null and alternative hypotheses (how to state).
- Details don't matter (each part of the hypothesis test must be met so this is not an issue - ie. the Law of Small Numbers).

Unit 8: Complex Numbers, Matrices, & Vectors

3 Days

Standards: **Red = key vocabulary Green = verb**

Learning Intention A:

Apply properties and operations with **complex numbers**.

- ❑ **NC.M4.N.1.1**: **Execute** procedures to add and subtract **complex numbers**.
- ❑ **NC.M4.N.1.2: Execute** procedures to multiply **complex numbers**.

Learning Intention B:

Apply properties and operations with **matrices** and **vectors**.

- ❑ **NC.M4.N.2.1**: **Execute** procedures of addition, subtraction, multiplication, and scalar multiplication on **matrices**.
- ❑ **NC.M4.N.2.2**: **Execute** procedures addition, subtraction, and scalar multiplication on **vectors**.

EQ and Unpacking	Vocabulary Black = New Gray = Review	Resources
EQ A: • Why are complex numbers necessary? • How is adding and subtracting complex numbers similar to combining like terms? • When multiplying complex numbers when will your product not contain an imaginary part? Explain. • What's different about adding complex numbers versus multiplying them? **Unpacking A:** • Simplify complex numbers using addition, subtraction, and multiplication. ** Please note that it is imperative to teach students conceptually the process of operating with complex numbers, but it is also important to teach students to use the TI- Graphing Calculator. While very basic questions can often be completed using a calculator, higher-level questions will use variables and abstract problems to see if students can find a concrete answer.*	Complex numbers a+bi, Conjugates real part imaginary part real numbers scalar **Element** **identity matrix** **Scalar**	• Vectors (Prezi) (Launch - Use Questions on Slides) • Textbook Pages for Resources (Launch Activity & Useful Notes/Examples) • N.1.1, N.1.2 Math is Fun • N.1.1, N.1.2 Computations with Complex Numbers Task • N.2.1 Khan Academy • N.2.1 Scalar and Matrix Multiplication (Purple Math) • N.2.1 Math is Fun • N.2.1Matrices Vocabulary Flashcards (Quizlet) • N.2.1 Intro to Matrices Video (MathisFun) • N.2.1Adding and Subtracting Matrices Worksheet • N.2.1 Scalar Multiplication Worksheet • N.2.1 UTEP Math Notes & Examples • N.2.1Multiplying Matrices Worksheet • N.2.2 Khan Academy • N.2.2 Vectors (MathisFun) • N.2.2 Intro to Vectors and Physics (YouTube) • N.2.2 Vectors Worksheets (Math Worksheet Land) • N.2.2 Scalar Multiplication f Vectors (Notes and Video)

EQ B: ● How do we add and subtract matrices? ● What is important to remember when multiplying matrices? Explain. ● How can we use the distributive property with scalar multiplication? ● What kinds of mathematical and real-world situations can be modeled by matrices? ● How is combining like terms similar to the process of adding and subtracting vectors? ● What kinds of mathematical and real-world situations can be modeled by vectors? ● What properties and operational understandings from the real number system apply to vectors? **Unpacking B:** ● Understand that the structure (rows and columns) of a matrix determines how matrices can be combined through addition, subtraction, and multiplication (including scalar multiplication) and use that structure to perform the aforementioned operations with matrices. ○ This understanding includes when operations on matrices cannot be completed. **Students are not expected to solve matrix equations.* ● Recognize the component form of a vector as $\boldsymbol{v} = \langle v_x, v_y \rangle$ where v_x and v_y represent the horizontal and vertical components, respectively. ● Be familiar with three different methods for adding and subtracting vectors: ● Adding vectors end-to-end - positioning the vectors (without changing their magnitudes and directions) so that the initial point of one vector coincides with the terminal point of the other vector ○ Adding/subtracting corresponding components - add or subtract the corresponding components ○ Using the parallelogram rule - a graphical method used for: ■ addition of two vectors, ■ subtraction of two vectors, and	**Magnitude** **Direction** **resultant** **vector**	Formative Assessment Lessons/Task ● **N1.1, N.1.2 Neat Matrix Multiplication Task** ● **N1.1, N.1.2 Complex Numbers** ● **N1.1, N.1.2 Add, Subtract, Multiply Complex Numbers** **Matrix Multiplication Task (possible homework assignment)** ● **N.2.1,N.2.2 Keeping Secrets (PBL unit on matrices)** ● **N.2.2 One Shot (PBL unit on parametric equations and vectors)** ● **N.1.1, N.1.2 Complex Number Patterns (Illustrative Mathematics)** ● **N.1.1, N.1.2 Vertex of a Parabola with Complex Roots (Illustrative Mathematics)** ● **N.2.1 Khan Academy Adding & Subtracting Practice** ● **N.2.1 Add & Subtract Matrices (IXL)** ● **N.2.1 Add & Subtract Scalar Matrices (IXL)** ● **N.2.1 Multiplying Matrices Practice (Khan Academy)** ● **N.2.1 Scalar Multiplication (IXL)** ● **N.2.1 Matrix Multiplication Assessments** ● **N.2.1 Matrix Multiplication Task (Open Middle)** ● **N.2.2 Vector Addition** ● **N.2.2 Vector Addition MATHEMATICAL VECTOR ADDITIONn Worksheet**

○ Resolution of a vector into two components in arbitrary directions. ● Understand the process of finding the sum/difference of two vectors using any of the methods mentioned. They are not expected to know the name of the methods. ● Apply the properties when multiplying a vector by a scalar. **Students are not expected to execute procedures for vectors beyond 2-dimensional vectors.*		